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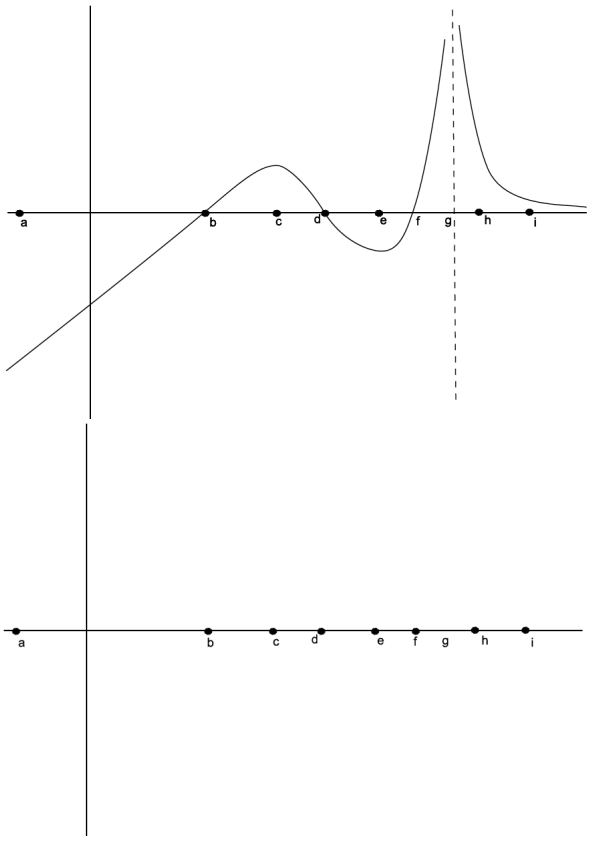
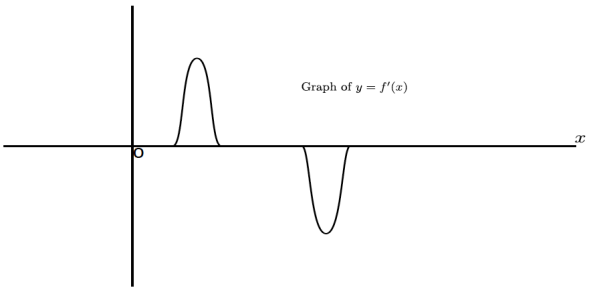
MAM1020S **Tutorial 6** **August 2017**

1. You are given that the function $y = f(x)$ has the graph below. On the given axes, sketch the graph of $y = f'(x)$ wherever it is defined. [Hint: Don't try to guess a formula for $f(x)$; there isn't one! At each of the indicated points, ask yourself more-or-less what $f'(x)$ is at that point; then fill in in a sensible way.]



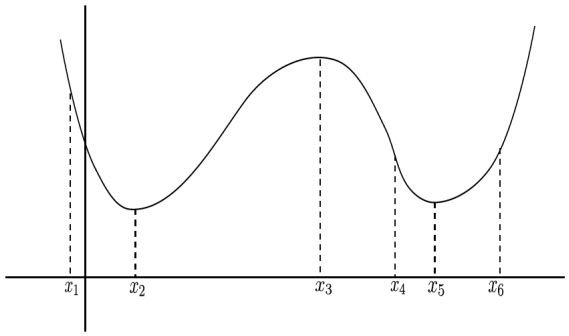
2. You are given below the graph of $f'(x)$ (not $f(x)$). Use the information to sketch a possible graph of $f(x)$. (Mostly the graph of f' runs along the x-axis with two "spikes".)



How many different sketches are possible? What is the same about all of them?

3. Let $H(x)$ be the height above sea-level, in metres, of the Gariep river at x kilometres from its source. What are the units of H'? What can you say about the sign of $H'(x)$?

4. The graph of f' (not f) is given below. At which of the marked x-values is (a) $f(x)$ greatest? (b) $f(x)$ least? (c) $f'(x)$ greatest? (d) $f'(x)$ least? (e) $f''(x)$ greatest? (f) $f''(x)$ least?



5. Show that if f is an even function then f' is an odd function. [Do this in two ways: think about the graph of an arbitrary even function and then imagine how the gradients of the tangents to points on the graph that are the same distance on either side of the y-axis will behave. This would be a "graphic" proof, but not very convincing algebraically. Secondly try a proof using the limit definition of $f'(x)$ and $f'(-x)$. Your proof must work for any even function; it is not good enough to check it for one even function.]

6. Show that if f is an odd function then f' is an even function.

7. For each function given below, calculate the derivative at a point $f'(a)$ using the derivative definition.

   (a) for the function defined by $f(x) = 2x^2 - 3x$ compute $f'(0)$.
   (b) for the function defined by $f(x) = \sqrt{2x+1}$ compute $f'(4)$.
   (c) for the function defined by $f(x) = \frac{1}{x-2}$ compute $f'(4)$.

8. For each function $f(x)$ given below, find the general derivative $f'(x)$ as a new function by using the derivative definition.

   (a) for the function defined by $f(x) = \sqrt{x-4}$ compute $f'(x)$.
   (b) for the function defined by $f(x) = \sqrt{-x^3}$ compute $f'(x)$.
   (c) for the function defined by $f(x) = \frac{x}{x+1}$ compute $f'(x)$.
   (d) for the function defined by $f(x) = \frac{1}{\sqrt{x}}$ compute $f'(x)$.

9. For each function $f(x)$ given below, find the equation of the tangent line at the indicated point using the derivative definition.

   (a) for the function defined by $f(x) = x - x^2$ compute the equation of the tangent line at $(2, -2)$.
   (b) for the function defined by $f(x) = 1 - 3x^2$ compute the equation of the tangent line at $(0, 1)$.
   (c) for the function defined by $f(x) = \frac{1}{2x}$ compute the equation of the tangent line at $x = 1$.
   (d) for the function defined by $f(x) = 1 + \sqrt{x}$ compute the equation of the tangent line at $x = 1$.